Calculus 1
Integration Review (2)

Name____________________
Date____________________

Integrate the following. Make sure you show all your work (show u du steps if necessary).

1) $\int\left(x^3 - 3x^2 + x + 4\right)dx$

$y = \frac{1}{4}x^4 - x^3 + \frac{1}{2}x^2 + 4x + c$

2) $\int\left(\frac{1}{4x^3} - \sqrt[5]{3x^2}\right)dx$

$\int\left(\frac{1}{4}x^{-3} - 3^{1/5}x^{2/5}\right)dx$

$\int\left(\frac{1}{4}x^{-3} - \sqrt[5]{3}x^{2/5}\right)dx$

$-\frac{1}{8}x^{-2} - \frac{5\sqrt[5]{3}}{7}x^{7/5} + c$

$-\frac{1}{8x^2} - \frac{5\sqrt[5]{3}\cdot\sqrt[5]{x^7}}{7} + c$

$-\frac{1}{8x^2} - \frac{5\sqrt[5]{3x^7}}{7} + c$

3) $\int\left(6x^2 + 8x\right)\left(x^3 + 2x^2\right)^4 dx$

$u = x^3 + 2x^2$

$\frac{du}{dx} = 3x^2 + 4x$

$\frac{du}{3x^2 + 4x} = dx$

$\int\left(6x^2 + 8x\right)u^4 \frac{1}{3x^2 + 4x}du$

$\int 2\left(3x^2 + 4x\right)u^4 \frac{1}{3x^2 + 4x}du$

$\int 2u^4 du$

$\frac{2}{5}u^5 + c$

$\frac{2}{5}(x^3 + 2x^2)^5 + c$

4) $\int \tan x \sec^2 x\, dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2(x)$

$\frac{du}{\sec^2(x)} = dx$

$\int u \sec^2 x \frac{du}{\sec^2 x}$

$\int u du$

$\frac{1}{2}u^2 + c$

$\frac{1}{2}\tan^2 x + c$

5) $\int \frac{1}{\sqrt{x}} sin(\sqrt{x})dx$

$\int x^{-1/2} \sin(x^{1/2})dx$

$u = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{\frac{1}{2}x^{-1/2}} = dx$

$\int x^{-1/2} \sin u \frac{du}{\frac{1}{2}x^{-1/2}}$

$\int 2\sin u du$

$-2\cos u + c$

$-2\cos(x^{1/2}) + c$

$-2\cos\left(\sqrt{x}\right) + x$

6) $\int \frac{x+2}{\sqrt{x^2+4x+7}}dx$

$\int (x+2)(x^2+4x+7)^{-1/2} dx$

$u = x^2 + 4x + 7$

$\frac{du}{dx} = 2x + 4$

$\frac{du}{2x+4} = dx$

$\int (x+2)u^{-1/2} \frac{du}{2x+4}$

$\int (x+2)u^{-1/2} \frac{du}{2(x+2)}$

$\int \frac{1}{2}u^{-1/2}du$

$u^{1/2} + c$

$(x^2+4x+7)^{1/2} + c$

$\sqrt{(x^2+4x+7)} + c$

7) $\int \left(6x^4 \sin^3(4x^5)\cos(4x^5)\right)dx$

$u = \sin(4x^5)$

$\frac{du}{dx} = \cos(4x^5)(20x^4)$

$\frac{du}{\cos(4x^5)(20x^4)} = dx$

$\int \left(6x^4u^3 \cos(4x^5)\right)\frac{du}{\cos(4x^5)(20x^4)}$

$\int \frac{6}{20}u^3 du$

$\int \frac{3}{10}u^3 du$

$\frac{3}{40}u^4 + c$

$\frac{3}{40}\sin^4(4x^5) + c$

8) $\int (3x+2)^2 dx$

$u = 3x + 2$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$\int (u)^2 \frac{du}{3}$

$\int \frac{1}{3}u^2 du$

$\frac{1}{9}u^3 + c$

$\frac{1}{9}(3x+2)^3 + c$

9) $\int (3x^2+2)^2 dx$

FOIL

$\int (9x^4 + 12x^2 + 4)dx$

$\frac{9}{5}x^5 + 4x^3 + 4x + c$

Solve for the constant of integration given the derivatives and a point of the original function.

10) $f'(x)=\int \sqrt[3]{x}\,dx \quad (1,2)$

$$\int dy=\int x^{1/3}dx$$
$$y=\frac{3}{4}x^{4/3}+c$$
$$2=\frac{3}{4}(1)^{4/3}+c$$
$$2=\frac{3}{4}+c$$
$$\frac{5}{4} \quad or \quad 1.25=c$$
$$y=\frac{3}{4}x^{4/3}+\frac{5}{4}$$
$$y=\frac{3}{4}\sqrt[3]{x^4}+\frac{5}{4}$$

11) $f'(x)=\int\left(\sec^2 x-\sin x\right)dx \quad \left(\frac{\pi}{4},1\right)$

$$\int dy=\int\left(\sec^2 x-\sin x\right)dx$$
$$y=\tan x+\cos x+c$$
$$1=\tan\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{4}\right)+c$$
$$c=-.707 \quad or \quad -\frac{\sqrt{2}}{2}$$
$$y=\tan x+\cos x-.707$$

12) $\frac{dy}{dx}=5x^{10}y^3 \quad (0,3)$

$$dy=5x^{10}y^3dx$$
$$\frac{dy}{y^3}=\frac{5x^{10}y^3dx}{y^3}$$
$$y^{-3}dy=5x^{10}dx$$
$$\int y^{-3}dy=\int 5x^{10}dx$$
$$-\frac{1}{2}y^{-2}=\frac{5}{11}x^{11}+c$$
$$-\frac{1}{2}(3)^{-2}=\frac{5}{11}(0)^{11}+c$$
$$-\frac{1}{2}\cdot\frac{1}{9}=c$$
$$-\frac{1}{18}=c$$
$$-\frac{1}{2}y^{-2}=\frac{5}{11}x^{11}-\frac{1}{18}$$